THE PERMANENT GRAVITATIONAL FIELD IN THE EINSTEIN THEORY.

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1. In accordance with the theory of Einstein a permanent gravitational field is defined by a quadratic differential form

$$ds^2 = \sum_{i,k}^{1,\ldots,4} g_{ik}dx_i dx_k \qquad (g_{ik} = g_{ki}), \tag{1}$$

where the g's, called the potentials of the field, are determined by the condition of satisfying ten partial differential equations of the second order, $G_{ik} = 0$. When the four coördinates x_i are functions of a single parameter, the locus of the point with these coördinates is a curve in four-space. If these functions are of such a character that the integral

$$\int \sqrt{\Sigma g_{ik}dx_i dx_k} \tag{2}$$

is stationary along the curve, the curve is called a "world-line," or a geodesic, in the four-space.

Einstein* considered the case when x_1, x_2, x_3, are rectangular coördinates and x_4 represents the time, and assumed that the field was produced by a mass at the origin which did not vary with the time. In order to obtain the equations of the world-lines in the form which enabled him to establish his well-known expression for the precession of the perihelion of Mercury, Einstein made also the following assumptions:

A. The quantities g are independent of t.

B. The equations $g_{i4} = 0$ for $i = 1, 2, 3$.

C. The solution is spacially symmetric with respect to the origin of coördinates in the sense that the solution is unaltered by an orthogonal transformation of x_1, x_2, x_3.

D. At infinity the quantities $g_{ik} = 0$ for $i \neq k$, and

$$g_{44} = -g_{11} = -g_{22} = -g_{33} = 1.$$

Schwarzschild† using the first three of these assumptions and certain others integrated the equations $G_{ik} = 0$, and obtained (1) in the form

* Berlin Sitzungsberichte, 1915, p. 831.

† Berlin Sitzungsberichte, 1916, p. 189.

$$(3)\qquad ds^2 = c^2\left(1 - \frac{\alpha}{R}\right)dt^2 - \frac{dR^2}{1 - \frac{\alpha}{R}} - R^2(d\theta^2 + \sin^2\theta d\varphi^2),$$

where α is a constant depending on the mass at the origin. Levi-Civita* has given three solutions of the equations $G_{ik} = 0$, one of which includes the above, and Weyl† has given still another solution. Later Kottler‡ obtained the form (3) not by the solution of the equations $G_{ik} = 0$ but as a consequence of certain postulates. It is the purpose of this paper to accomplish the same result by the following set of postulates:

I. Assumptions A and B of Einstein, in accordance with which we write (1) in the form

$$(4)\qquad ds^2 = V^2dt^2 - ds_0^2,$$

where

$$(5)\qquad ds_0^2 = \sum_{i,k}^{1,2,3} a_{ik}dx_idx_k,$$

the functions V and a_{ik} being independent of t.

II. The function V is a solution of

$$\Delta_2\theta = 0,$$

where $\Delta_2\theta$ is the Beltrami differential parameter formed with respect to the form (5), and is defined by

$$(6)\qquad \Delta_2\theta = \frac{1}{\sqrt{a}}\sum_i \frac{\partial}{\partial x_i}\left(\sum_k a^{(ik)}\sqrt{a}\frac{\partial\theta}{\partial x_k}\right),$$

where a is the determinant of the functions a_{ik} and $a^{(ik)}$ is the cofactor of a in this determinant divided by a.§ This assumption is equivalent to the equation $G_{44} = 0$. In this equation and hereafter $\sum_j$ means the sum for $j = 1, 2, 3$.

III. The surfaces $V =$ const. form part of a triply orthogonal system in the space, S_3, of coördinates x_1, x_2, x_3.

IV. The orthogonal trajectories of $V =$ const. in S_3 are paths of the particle, in the sense that the coördinates x_1, x_2, x_3, of a world-line determine a path in S_3 of a particle in the gravitational field for which the world-line is the representation in terms of space and time t.

V. The form (5) is euclidean to a first approximation.

2. Geodesics in the four-space and in S_3. It can be shown|| that in any three-space there exist triply-orthogonal systems of surfaces, and accord-

* Rendiconti dei Lincei, ser. 5, vol. 27 (1918), p. 365.

† Ann. der Physik, vol. 54 (1917), p. 117.

‡ Ann. der Physik, vol. 56 (1918), p. 401.

§ Bianchi, Lezioni di Geometrica Differenziale, 2d ed., vol. 1, p. 68.

|| Wright, Invariants of Quadratic Differential Forms, Cambridge Tract No. 9, pp. 64–67.

ingly (5) can be given the form

$$(7) \qquad ds_0^2 = \sum_i a_i dx_i^2,$$

where now the coördinate surfaces form a triply orthogonal system.

If we take s for the parameter along a world-line, and put $\dot{x}_i = dx_i/ds$, $\dot{t} = dt/ds$, the integral (2) becomes

$$(8) \qquad \int \sqrt{V^2\dot{t}^2 - \Sigma a_i \dot{x}_i^2}\, ds \equiv \int \varphi(V, x_1, x_2, x_3, \dot{x}_1, \dot{x}_2, \dot{x}_3, \dot{t})\, ds.$$

The Euler equations of condition that (8) be stationary are

$$(9) \qquad \frac{\partial \varphi}{\partial x_i} - \frac{d}{ds}\left(\frac{\partial \varphi}{\partial \dot{x}_i}\right) = 0^* \qquad (i = 1, 2, 3, 4;\ x_4 = t).$$

Applying these conditions to (8), and noting that in consequence of the choice of s as parameter, we have $\varphi = 1$ along a world-line, we obtain

$$(10) \quad \frac{d^2x_i}{ds^2} + \sum_j \frac{\partial \log a_i}{\partial x_j}\frac{dx_i}{ds}\frac{dx_j}{ds} - \frac{1}{2a_i}\sum_j \frac{\partial a_j}{\partial x_i}\left(\frac{dx_j}{ds}\right)^2 + \frac{V}{a_i}\frac{\partial V}{\partial x_i}\left(\frac{dt}{ds}\right)^2 = 0,$$

$$(11) \qquad \frac{dt}{ds} = \frac{k^2}{V^2},$$

where k is a constant.

By definition the geodesics is S_3 are the curves along which the integral $\int \sqrt{\Sigma a_i dx_i^2}$ is stationary. When s_0 is taken for the parameter along such a geodesic, we find that the equations of a geodesic are

$$(12) \qquad \frac{d^2x_i}{ds_0^2} + \sum_j \frac{\partial \log a_i}{\partial x_j}\frac{dx_i}{ds_0}\frac{dx_j}{ds_0} - \frac{1}{2a_i}\sum_j \frac{\partial a_j}{\partial x_i}\left(\frac{dx_j}{ds_0}\right)^2 = 0.$$

From (4) and (11) it follows that the parameters s and s_0 along a world-line and the corresponding path in S_3 are in the relation

$$(13) \qquad ds_0 = \sqrt{\frac{k^2}{V^2} - 1}\, ds.$$

When we express equations (10) in terms of s_0, we obtain

$$(14) \qquad \begin{aligned} &\frac{d^2x_i}{ds_0^2} + \sum_j \frac{\partial \log a_i}{\partial x_j}\frac{dx_i}{ds_0}\frac{dx_j}{ds_0} - \frac{1}{2a_i}\sum_j \frac{\partial a_j}{\partial x_i}\left(\frac{dx_j}{ds_0}\right)^2 \\ &\qquad = \frac{k^2}{V(k^2 - V^2)}\left(\frac{dV}{ds_0}\frac{dx_i}{ds_0} - \frac{1}{a_i}\frac{\partial V}{\partial x_i}\right). \end{aligned}$$

From this equation and (12) it follows that a necessary condition that the path of a particle be a geodesic in S_3 is

* Bolza, Lectures on the Calculus of Variations, p. 123.

$$\frac{dV}{ds_0}\frac{dx_i}{ds_0} = \frac{1}{a_i}\frac{\partial V}{\partial x_i} \qquad (i = 1, 2, 3). \tag{15}$$

If we multiply these respective equations by $\sqrt{a_i}$, square the resulting equations and add them, we get

$$\left(\frac{dV}{ds_0}\right)^2 = \sum_i \frac{1}{a_i}\left(\frac{\partial V}{\partial x_i}\right)^2 \equiv \Delta_1 V, \tag{16}$$

where $\Delta_1\theta$ is the first differential parameter of θ with respect to the form (7). When ds_0^2 is written in the general form (5), the expression for $\Delta_1\theta$ is

$$\Delta_1\theta = \sum_{ik} a^{(ik)}\frac{\partial V}{\partial x_i}\frac{\partial V}{\partial x_k}.* \tag{17}$$

Hence (15) may be written in the form

$$\frac{dx_i}{ds_0} = \frac{1}{\sqrt{\Delta_1 V}}\cdot\frac{1}{a_i}\frac{\partial V}{\partial x_i}. \tag{18}$$

The direction of the tangent to any curve on a surface $V =$ const. through the point P where a path curve meets the surface is given by the values of dx_i/ds_1, s_1 being the arc along the curve, where

$$\sum_i \frac{\partial V}{\partial x_i}\frac{dx_i}{ds_1} = 0. \tag{19}$$

From (18) and (19) it follows that

$$\Sigma a_i \frac{dx_i}{ds_0}\frac{dx_i}{ds_1} = 0, \tag{20}$$

which is the condition that the path is orthogonal to the surface, since this condition is satisfied by every curve through P.†

Conversely, any orthogonal trajectory of the surfaces $V =$ const. is defined by (18). In fact from (20) and the equation

$$\Sigma a_i \frac{dx_i}{ds_0}\frac{dx_i}{ds_2} = 0,$$

for a second curve on $V =$ const. we get

$$\frac{\dfrac{dx_1}{ds_0}}{a_2a_3\left(\dfrac{dx_2}{ds_1}\dfrac{dx_3}{ds_2} - \dfrac{dx_3}{ds_1}\dfrac{dx_2}{ds_2}\right)} = \cdots = \frac{\dfrac{dx_3}{ds_0}}{a_1a_2\left(\dfrac{dx_1}{ds_1}\dfrac{dx_2}{ds_2} - \dfrac{dx_2}{ds_1}\dfrac{dx_1}{ds_2}\right)} = R, \tag{21}$$

* Bianchi, l. c., p. 61.

† Bianchi, l. c., p. 330.

where by composition we find

$$R = \frac{1}{\sqrt{a_1 a_2 a_3 Q}}, \qquad Q = \sqrt{\Sigma a_2 a_3 \left(\frac{dx_2}{ds_1}\frac{dx_3}{ds_2} - \frac{dx_3}{ds_1}\frac{dx_2}{ds_2} \right)^2}.$$

In like manner from (19) and

$$\Sigma \frac{\partial V}{\partial x_i} \frac{dx_i}{ds_2} = 0,$$

we obtain

$$(22) \quad \frac{\dfrac{\partial V}{\partial x_1}}{\dfrac{dx_2}{ds_1}\dfrac{dx_3}{ds_2} - \dfrac{dx_3}{ds_1}\dfrac{dx_2}{ds_2}} = \cdots = \frac{\dfrac{\partial V}{\partial x_3}}{\dfrac{dx_1}{ds_1}\dfrac{dx_2}{ds_2} - \dfrac{dx_2}{ds_1}\dfrac{dx_1}{ds_2}} = \frac{\sqrt{a_1 a_2 a_3}\sqrt{\Delta_1 V}}{Q}.$$

From (21) and (22) follows (18).

The above results may be stated as follows:

If the path of a particle in a permanent gravitational field is a geodesic, it is an orthogonal trajectory of the surfaces $V = const.$

3. **Condition that the orthogonal trajectories of the surfaces V = const. be geodesics.** In establishing this condition we make use of the mixed differential parameter of the first order, $\Delta_1(\theta, \varphi)$, which when formed with respect to (5) is defined by

$$\Delta_1(\theta, \varphi) = \sum_{i, k} a^{(ik)} \frac{\partial \theta}{\partial x_i} \frac{\partial \varphi}{\partial x_k}.*$$

For the form (7) this is

$$\Delta_1(\theta, \varphi) = \sum_i \frac{1}{a_i} \frac{\partial \theta}{\partial x_i} \frac{\partial \varphi}{\partial x_i}.$$

From (18) we have

$$\frac{d^2 x_i}{ds_0^2} = \sum_j \frac{\partial}{\partial x_j}\left(\frac{1}{\sqrt{\Delta_1 V}} \frac{1}{a_i} \frac{\partial V}{\partial x_i} \right) \cdot \frac{1}{\sqrt{\Delta_1 V}} \frac{1}{a_j} \frac{\partial V}{\partial x_j} = \frac{1}{\sqrt{\Delta_1 V}} \Delta_1 \left(V, \frac{1}{\sqrt{\Delta_1 V}} \frac{1}{a_i} \frac{\partial V}{\partial x_i} \right).$$

Substituting in (12), we get

$$a_i \sqrt{\Delta_1 V} \Delta_1 \left(V, \frac{1}{\sqrt{\Delta_1 V}} \frac{1}{a_i} \frac{\partial V}{\partial x_i} \right) \sum_j \frac{1}{a_j} \frac{\partial \log a_i}{\partial x_j} \frac{\partial V}{\partial x_i} \frac{\partial V}{\partial x_j} - \frac{1}{2} \sum_j \frac{1}{a_j} \frac{\partial \log a_j}{\partial x_i} \left(\frac{\partial V}{\partial x_j} \right)^2 = 0,$$

which may be written

$$(23) \quad \Delta_1 \left(V, \frac{\partial V}{\partial x_i} \right) - \frac{1}{2} \frac{\partial V}{\partial x_i} \frac{\Delta_1 (V, \Delta_1 V)}{\Delta_1 V} - \frac{1}{2} \sum_j \frac{1}{a_j} \frac{\partial \log a_j}{\partial x_i} \left(\frac{\partial V}{\partial x_j} \right)^2 = 0,$$

since

* Bianchi, Lezioni, p. 61.

$$(24) \qquad \Delta_1(V,\, \Delta_1 V) = \sum_j \frac{2}{a_j}\, \Delta_1\left(V, \frac{\partial V}{\partial x_j}\right)\frac{\partial V}{\partial x_j} + \sum_j \left(\frac{\partial V}{\partial x_j}\right)^2 \Delta_1\left(V, \frac{1}{a_j}\right).$$

Since

$$\frac{1}{2}\frac{\partial}{\partial x_i}\, \Delta_1 V = \Delta_1\left(V, \frac{\partial V}{\partial x_i}\right) - \frac{1}{2}\sum_j \frac{1}{a_j}\frac{\partial \log a_j}{\partial x_i}\left(\frac{\partial V}{\partial x_j}\right)^2,$$

equation (23) may be written

$$\frac{1}{2}\frac{\partial}{\partial x_i}(\Delta_1 V)^2 = \Delta_1(V,\, \Delta_1 V)\frac{\partial V}{\partial x_i}.$$

Consequently $\Delta_1(V, \Delta_1 V)$, and also $\Delta_1 V$, must be functions of V. But when $\Delta_1 V$ is a function of V so also is $\Delta_1(V, \Delta_1 V)$. Hence we have the theorem:

A necessary and sufficient condition that the orthogonal trajectories of the surfaces V = const. be geodesics is that ΔV be a function of V.

In this case the surfaces V = const. are said to form a *geodesically parallel* family.*

4. **The path of a ray of light.** The function V is interpreted as the velocity of light in the field, and consequently along a world-line of a ray of light $ds = 0$, as follows from (4). In order to obtain the equations of these world-lines, we apply the Fermat principle that $\int dt$ be stationary along such a line, that is the integral $\int \sqrt{V^{-2}\Sigma a_i dx_i^2}$. This gives the equations

$$\frac{d^2x_i}{dt^2} + \frac{dx_i}{dt}\sum_j \frac{\partial}{\partial x_j}\log\frac{a_i}{V^2}\frac{dx_j}{dt} - \frac{V^2}{2a_i}\sum_j \frac{\partial}{\partial x_i}\frac{a_j}{V^2}\left(\frac{dx_j}{dt}\right)^2 = 0.$$

When we require that a path of light be a geodesic in S_3, we obtain (18). Hence:

When the orthogonal trajectories of the surfaces V = const. are paths of a particle in a permanent gravitational field, they are also the paths of a ray of light, and conversely.

5. **Certain triply orthogonal systems in euclidean space.** A necessary and sufficient condition that

$$ds^2 = H_1^2 dx_1^2 + H_2^2 dx_2^2 + H_3^2 dx_3^2$$

the linear element of euclidean space is that the functions H_i satisfy the following equations of Lame:†

* For other proofs of this theorem the reader is referred to Bianchi, l. c., p. 338; also, Wright, l. c., p. 64.

† Eisenhart, Differential Geometry, p. 449.

$$(25)\qquad \begin{gathered} \frac{\partial^2 H_i}{\partial x_j \partial x_k} = \frac{1}{H_j}\frac{\partial H_j}{\partial x_k}\frac{\partial H_i}{\partial x_j} + \frac{1}{H_k}\frac{\partial H_k}{\partial x_j}\frac{\partial H_i}{\partial x_k}, \\ \frac{\partial}{\partial x_i}\left(\frac{1}{H_i}\frac{\partial H_j}{\partial x_i}\right) + \frac{\partial}{\partial x_j}\left(\frac{1}{H_j}\frac{\partial H_i}{\partial x_j}\right) + \frac{1}{H_k^2}\frac{\partial H_i}{\partial x_k}\frac{\partial H_j}{\partial x_k} = 0, \end{gathered}$$

where $i \neq j$, $i \neq k$, $j \neq k$.

We consider the case where H_1 is a function of x_1 alone, and write

$$(26)\qquad H_1 = X_1',$$

the accent indicating differentiation with respect to x_1. Now equations (25) reduce to

$$(27)\qquad \frac{\partial}{\partial x_1}\left(\frac{1}{H_3}\frac{\partial H_2}{\partial x_3}\right) = 0, \qquad \frac{\partial}{\partial x_1}\left(\frac{1}{H_2}\frac{\partial H_3}{\partial x_2}\right) = 0,$$

$$(28)\qquad \frac{\partial}{\partial x_1}\left(\frac{1}{H_1}\frac{\partial H_2}{\partial x_1}\right) = 0, \qquad \frac{\partial}{\partial x_1}\left(\frac{1}{H_1}\frac{\partial H_3}{\partial x_1}\right) = 0,$$

$$(29)\qquad \frac{\partial}{\partial x_2}\left(\frac{1}{H_2}\frac{\partial H_3}{\partial x_2}\right) + \frac{\partial}{\partial x_3}\left(\frac{1}{H_3}\frac{\partial H_2}{\partial x_3}\right) + \frac{1}{H_1^2}\frac{\partial H_2}{\partial x_1}\frac{\partial H_3}{\partial x_1} = 0.$$

From (28) we have by integration

$$(30)\qquad H_2 = \sigma X_1 + \bar{\sigma}, \qquad H_3 = \tau X_1 + \bar{\tau},$$

where σ, $\bar{\sigma}$, τ and $\bar{\tau}$ are independent of x_1. In accordance with (27) these functions must satisfy the conditions

$$\tau\frac{\partial \bar{\sigma}}{\partial x_3} - \bar{\tau}\frac{\partial \sigma}{\partial x_3} = 0, \qquad \sigma\frac{\partial \bar{\tau}}{\partial x_2} - \bar{\sigma}\frac{\partial \tau}{\partial x_2} = 0.$$

If we replace these equations by

$$(31)\qquad \begin{gathered} \frac{\partial \bar{\sigma}}{\partial x_3} = \lambda\bar{\tau}, \qquad \frac{\partial \sigma}{\partial x_3} = \lambda\tau, \\ \frac{\partial \bar{\tau}}{\partial x_2} = \mu\bar{\sigma}, \qquad \frac{\partial \tau}{\partial x_2} = \mu\sigma, \end{gathered}$$

we have

$$\frac{1}{H_3}\frac{\partial H_2}{\partial x_3} = \lambda = \frac{1}{\tau}\frac{\partial \sigma}{\partial x_3}, \qquad \frac{1}{H_2}\frac{\partial H_3}{\partial x_2} = \mu = \frac{1}{\sigma}\frac{\partial \tau}{\partial x_2},$$

so that (29) becomes

$$(32)\qquad \frac{\partial}{\partial x_2}\left(\frac{1}{\sigma}\frac{\partial \tau}{\partial x_2}\right) + \frac{\partial}{\partial x_3}\left(\frac{1}{\tau}\frac{\partial \sigma}{\partial x_3}\right) + \sigma\tau = 0.$$

Each solution of this equation determines λ and μ, and then $\bar{\sigma}$ and $\bar{\tau}$ follow from (31).

Equation (32) expresses the condition that

$$\sigma^2 dx_2^2 + \tau^2 dx_3^2$$

is the linear element of the unit sphere.*

If it is required further that $H_2 H_3$ be the product of a function of x_1 alone, say φ, and a function ψ independent of x_1, we have, from (30)

$$\frac{\sigma\tau}{\psi} X_1^2 + \frac{\sigma\bar{\tau} + \bar{\sigma}\tau}{\psi} X_1 + \frac{\bar{\sigma}\bar{\tau}}{\psi} = \varphi.$$

From the two equations obtained by differentiating this equation with respect to x_2 and x_3 and (31) we find that $\sigma/\bar{\sigma} = \tau/\bar{\tau} = k$, where k is a constants. Consequently the general solution may be written

$$H_2 = X_1\sigma, \qquad H_3 = X_1\tau. \tag{33}$$

6. **Derivation of the Schwarzschild form (3).** In accordance with postulate III, we take the surfaces $V =$ const. for the coördinate surfaces $x_1 =$ const. of a triply orthogonal system in S_3, and write

$$V^2 = c^2(1 + 2\varphi_1(x_1)), \tag{34}$$

where c is the constant velocity of light.

From the results of §§ 2, 3 it follows that postulate IV is equivalent to $\Delta_1 V = \varphi(x_1)$, which reduces to

$$a_1 = \frac{V'^2}{\varphi} = \frac{c^2 \varphi_1'^2}{\varphi(1 + 2\varphi_1)}. \tag{35}$$

Since by postulate II we have $\Delta_2 V = 0$, we must have also

$$a_2 a_3 = \frac{\psi^2}{\varphi}, \tag{36}$$

where ψ is independent of x_1.

From the results of § 5 it follows that the linear element of euclidean space, satisfying the conditions that a_1 is a function of x_1 alone and (36), can be given the form

$$d\bar{s}^2 = dx_1^2 + x_1^2(\sigma^2 dx_2^2 + \tau^2 dx_3^2). \tag{37}$$

In accordance with postulate V the linear element of S_3 is to be (37) to a first approximation. From (35) and (36) we find that such an approximation is given by taking

$$\varphi_1'^2 = \frac{\varphi}{c^2}, \qquad \varphi = \frac{m^2}{4x_1^4}, \tag{38}$$

* Eisenhart, Differential Geometry, p. 157.

where m is a constant, that is $\varphi_1 = -m/2cx_1$. Then from (34), (35) and (37) we have

$$(39)\qquad ds^2 = c^2\left(1 - \frac{m}{cx_1}\right)dt^2 - \frac{1}{1 - \frac{m}{cx_1}}dx_1^2 - x_1^2(\sigma^2dx_2^2 + \tau^2dx_3^2).$$

When we take the solutions $\sigma = 1$, $\tau = \sin x_2$ of (32), we obtain (3).

7. **Derivation of a form due to Levi-Civita.** If in (26) and (33) we put $X_1 = 1/\sqrt{K_0\mu}\,x_1$, where K_0 and μ are constants, we have in place of (37)

$$d\bar{s}^2 = \frac{1}{K_0\mu x_1^4}dx_1^2 + \frac{1}{x_1^2}\left(\frac{\sigma^2dx_2^2 + \tau^2dx_3^2}{K_0\mu}\right),$$

and the expression in parenthesis has curvature $K_0\mu$. In place of (34) we put

$$V^2 = \mu V_0^2(1 + 2\varphi_1(x_1)),$$

where V_0 is a constant. Proceeding as in the preceding section we have in place of (38)

$$\frac{V_0^2\varphi_1'^2}{\varphi} = \frac{1}{K_0x_1^4\mu^2}, \qquad \varphi = mx_1^4,$$

where m is a constant. If we take accordingly

$$2\varphi_1 = -\frac{\epsilon x_1}{\mu}, \qquad m = \frac{V_0^2K_0}{4}, \qquad \epsilon = \pm 1,$$

we obtain the form

$$ds^2 = V_0^2(\mu - \epsilon x_1)dt^2 - \frac{dx_1^2}{K_0x_1^4(\mu - \epsilon x_1)} - \frac{1}{x_1^2}\left(\frac{\sigma^2dx_2^2 + \tau^2dx_3^2}{K_0\mu}\right),$$

due to Levi-Civita.*

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* L. c.

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